Probability and Decision Making

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Definitions

- Probability
  - Likelihood of an event
  - Number of successful events / total number of equally likely possible events
- Decision making
  - Choices that affect the possible outcome of a game

Basic Rules

- Probability of an event is between 0 and 1, or 0% and 100%, inclusive
- Convert from decimal to percentage by moving decimal point 2 spaces to the right
  - 0.125 = 12.5%
  - 50% = .50
- Sum of all possible events = 100%

Basic Calculations

- To calculate the probability of an event
  - Determine the total number of possible equally likely events (N)
  - Determine the number of successful events (S)
  - Probability of success = S / N
  - Example: what is the probability of rolling a 1 on a die?

What Type of Die?

- Ordinary 6-sided die
  - 6 equally likely possible events (1,2,3,4,5,6)
  - 1 successful event (1)
  - Pr(1) = 1/6 = 16.7%
- Ordinary 4-sided die
  - 4 equally likely possible events (1,2,3,4)
  - 1 successful event (1)
  - Pr(1)= ¼ = 25%

Other Dice

- 4-sided Formula De die
  - 4 equally likely possible events (1,1,2,2)
  - 2 successful events (1,1)
  - Pr(1) = 2/4 = 50%
- 6-sided Formula De die
  - 6 equally likely possible events (2,3,3,4,4,4)
  - 0 successful events
  - Pr(1) = 0/6 = 0%

Multiple Events

- “Or” vs. “And”
  - Or: at least one of the events must happen
  - And: all of the events must happen
- “Or” means add
- “And” means multiply

Or

- What is the probability of rolling a 1 or a 2 on a normal 6-sided die?
  - 6 equally likely possible events (1,2,3,4,5,6)
  - 2 successful events (1,2)
  - Pr (1 or 2) = 2/6 = 33.3%
- Alternative method
  - Pr(1) + Pr(2) = 16.7% + 16.7% = 33.3% (*)
  - This method requires an additional step if the successful events are not disjoint

Or (cont.)

- What is the probability of rolling less than 3 or odd on a normal 6-sided die?
  - Basic method
    - 6 equally likely possible events
    - 4 successful events (1,2,3,5)
    - Pr(<3 or odd) = 4/6 = 66.7%
  - Alternative method
    - Pr(<3) + Pr(odd) = 33.3% + 50% = 83.3%
    - Need to adjust for overlap (<3 and odd)

And

- What is the probability of rolling a 1 on each of 2 dice?
  - 36 equally likely possible events
  - 1 successful event (1-1)
  - Pr(two 1s) = 1/36 = 2.8%
- Alternative method
  - Pr(1) x Pr(1) = 16.7% x 16.7% = 2.8%
  - This method requires adjustment if events are dependent

Independent vs. Dependent Events

- The outcome of two events are independent if the fact that one event happens does not affect the probability of the second event happening.
- Drawing without replacement
  - 3 of 16 grey tiles in War of the Rings are “3”s
  - Pr(3) on each draw is 3/16
  - After 1st draw, Pr(3) on 2nd draw is 2/15 or 3/15, depending upon the result of the first draw.
  - These are contingent probabilities

Contingent Probability

- Probability of a second event given that a first event happened
- If the outcome of two events are independent, the contingent probability is the same as the regular probability
- Pr(3 on 2^{nd}| 3 on 1^{st}) = 2/15 = 13.3%
- Pr(3 on 1^{st} and 3 on 2^{nd}) = Pr(3 on 1^{st}) x Pr(3 on 2^{nd}| 3 on 1^{st}) = 3/16 x 2/15 = 2.5%

Disjoint Events

- Events are disjoint if at most one of them can happen
  - Bruce Reiff winning Auction and Pro Golf is not disjoint – he can win both
  - Bruce Reiff and Arthur Field winning Battleline is disjoint – at most one of them can happen
- If events are disjoint, the probability of at least one happening (“Or”) is equal to the sum of the probabilities of either event happening

Not Disjoint Events

- If events are not disjoint, the probability of at least one event happening is equal to the sum of the probabilities of either event happening, less the probability of both happening
- Pr(<3 or odd) = Pr(<3) + Pr(odd) – Pr(<3 and odd) = 33.3% + 50% - 16.7% = 66.7%

Dice

- Dice
  - Standard 6-sided dice
    - 1/6 chance of each result
  - Standard N-sided dice
    - 1/N chance of each result
  - Dice with more than 1 side with same result
    - S/N chance
  - Loaded dice
    - Can test empirically
  - Sum of dice

Other Random Number Generators

- Spinners
  - Ratio of edge of region to circumference
  - Ratio of area of region to area of circle
- Drawing with or without replacement
  - Tiles
  - Cards
  - Chits
- Simultaneous decisions
  - Rock, paper, scissors
  - Selection of numbers, add and mod

Multiple Events

- The expected number of successful events is equal to the probability of a successful event times the number of attempts
- The probability of at least one successful event is (generally) less than the probability of a successful event times the number of attempts
  - The lower the probability of a successful event, the closer the probability of at least one successful event is to the product

Extra Knowledge

- Reading your opponent
- Peeking
- Bent or marked cards
- Remember Guys and Dolls

Expected Value

- Sum of probability of each event times the value of each event
  - Expected value of the roll of one die

1/6 x 1 = 1/6

1/6 x 2 = 2/6

1/6 x 3 = 3/6

1/6 x 4 = 4/6

1/6 x 5 = 5/6

1/6 x 6 = 6/6

Total 21/6 = 3.5

Expected Value (cont.)

- Expected value does not have to equal any of the possible outcomes
- Expected value will be between highest and lowest possible outcomes
- Typically should select option that provides highest expected value
- May be difficult to identify value of each outcome to be used in calculation

Types of Outcomes

- Specific event
  - Ex. - Add 1 to the roll or add a die
- Assets
  - Find the key asset for comparison
  - Ex. – GOA: money or actions
- Victory points
- Winning

Phase of Game

- Early – assets
- Middle – victory points
- Endgame – winning
  - Margin of victory (or loss) vs. chance of winning
  - America's Cup

Game Theory

- Generally applies to simultaneous decision making
- Sequential but hidden decision making is equivalent
- Assume opponent picks optimal strategy
- If one strategy always yields better outcome, select that strategy (pure strategy)
- Otherwise, randomly select strategy, but weighted towards more favorable outcome (mixed strategy)
- Minmax

Fog of War

- Troop location
- Hidden cards
- Simultaneous movement

Two-player vs. Multi-player

- Cooperation vs. competition
- Two-player games are inherently zero sum
  - Metagaming may change this
- Multi-player games typically are not zero sum
  - Trading
  - Attack vs. growth
  - Multiple winners

Law of Large Numbers

- The more random events, the closer the average of the events is likely to be to the expected value
- Typically, the more random events in a game, the less impact luck has on the game

Card Counting

- Determine what is important
- Know the game components
  - Of little or no value if you do not know the starting composition
- Figure out what is left
  - Even if you can remember every card played, you may not be able to apply that knowledge